MATH5010 Linear Analysis (2022-22): Homework 2. Deadline: 1 Oct 2022

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Show that the finite sequence space c_{00} is not a closed subspace of ℓ_1 under the $\|\cdot\|_1$-norm.

2. Let X and Y be normed space. Define $X \times Y := \{(x, y) : x \in X; y \in Y$. The space $X \times Y$ is endowed the norm by $\|(x, y)\|_0 := \|x\|_X + \|y\|_Y$ for $(x, y) \in X \times Y$.
   Let $f : X \to Y$ be a mapping let $G(f) := \{(x, f(x)) : x \in X\}$ (called the *graph* of f).

   (i) Show that if f is a continuous mapping on X, then the graph $G(f)$ is a closed subset of $X \times Y$ under the norm $\|\cdot\|_0$.

   (ii) Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } \ x > 0; \\ 0 & \text{if } \ x < 0 \\ 1 & \text{if } x = 0. \end{cases}$$

   Show that the graph of f is not a closed subset of $\mathbb{R} \times \mathbb{R}$ under the norm $\|\cdot\|_0$ defined as above.

$* * *$ **End** $* * *$